ÇANKAYA UNIVERSITY

Department of Mathematics and Computer Science

MCS 205 - Basic Linear Algebra

SECOND MIDTERM EXAMINATION
17.12.2015

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR:

DURATION: 110 minutes

Question	Grade	Out of
1		25
2		20
3		25
4		20
5		20
Total		110

IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 5 problems.

3) Show all your work. No points will be given to correct answers without reasonable work.

Question 1. Let W be the subspace of $\mathbb{R}^3$ spanned by the following vectors

$$v_1 = (1, 4, -2), \quad v_2 = (2, -3, 1), \quad v_3 = (-8, 12, -4), \quad v_4 = (1, 37, -17)$$

(a) Find a basis S for W.

(b) Express each vector v_i not in S as a linear combination of the vectors in S.

(c) Find a basis T for $\mathbb{R}^3$ containing S.

Question 2. Let

$$A = \begin{bmatrix} 1 & 3 & -2 & -5 & 2 & 1 \\ 3 & 9 & -5 & -13 & 6 & 3 \\ -2 & -6 & 8 & 18 & -4 & 1 \end{bmatrix}$$

(a) Find a basis for the row space of A.

(b) Find a basis for the column space of A.

(c) Find a basis for the null space of A.

(d) Find the rank and the nullity of A.

Question 3.

(a) Consider the basis $S = \{(1,0,1),\ (0,1,-2),\ (0,1,3)\}$ and $T = \{(2,1,0),\ (1,0,3),\ (0,1,0)\}$.

i) Find a transition matrix Q from T to S.

ii) If $[v]_T = \begin{bmatrix} 3 \\ 2 \\ -7 \end{bmatrix}$, find $[v]_S$ by using the transition matrix in (i).

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(b) Given ordered bases $C = \{u_1, u_2, u_3\}$, $D = \{w_1, w_2, w_3\}$ with the transtion matrix

$$P_{D \leftarrow C} = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & -3 \\ 2 & 5 & -4 \end{bmatrix}$$

Write down the vector $2u_1 - 3u_2 + u_3$ as a linear combination of w_1, w_2 and w_3.

Question 4.)

(a) Let $S = \{v_1,\ v_2,\ v_3\}$ be a basis for the finite dimensional vector space V and $B = \{u_1,\ u_2,\ u_3\}$, $C = \{w_1,\ w_2,\ w_3\}$ be subsets of V where

$$\begin{aligned} u_1 &= v_1 + v_2 & \qquad w_1 &= v_1 + v_2 \\ u_2 &= v_1 + 2v_2 + v_3 & w_2 &= v_1 - v_2 \\ u_3 &= 3v_2 - v_3 & w_3 &= -v_1 - v_2 + v_3 \end{aligned}$$

i) Show that B is linearly independent.

ii) Show that C spans V.

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(b) Let $\{u_1, u_2, u_3, u_4\}$ be a basis for a vector space V. Find dimension of U where

$$U = \text{span}\{u_1 + 2u_2 + u_3 + u_4,\ u_1 + 3u_2 + u_3 + 2u_4,\ 3u_1 + 4u_2 + 2u_3,\ 3u_1 + 5u_2 + 2u_3 + u_4\}.$$

Question 5.

(a) Let $M_{2\times 2}$ be the vector space of all 2×2 matrices whose entries are real numbers. Let

$$W = \{A \in M_{2\times 2} \ : \ A^T = -A\}.$$

i) Show that W a subspace of $M_{2\times 2}$.

ii) Find a basis for W. What is the dimension of W?

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(b) Let $W = \left\{ A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} : ad = 0 \right\}$. Is W a subspace of $M_{2\times 2}$?