ÇANKAYA UNIVERSITY

Department of Mathematics and Computer Science

MCS 205 - Basic Linear Algebra

FIRST MIDTERM EXAMINATION
19.11.2015

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR:

DURATION: 100 minutes

Question	Grade	Out of
1		20
2		20
3		20
4		20
5		25
Total		105

IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 5 problems.

3) Show all your work. No points will be given to correct answers without reasonable work.

1) Consider the linear system

$$\begin{aligned} 2x_1 + 4x_2 + 6x_3 + 5x_4 - x_5 &= 0 \\ x_1 + 2x_2 + 2x_3 + x_4 &= 0 \\ 2x_1 + 4x_2 + 4x_4 &= 6 \\ 3x_1 + 6x_2 - 6x_4 - x_5 &= -4 \end{aligned}$$

(a) Form the augmented matrix of the given system.

(b) Find the (reduced) row echelon form of the augmented matrix, which is found in part (a).

(c) Solve the system if it is consistent.

2) Find $a, b \in \mathbb{R}$ for which the following system has,

(a) no solution, **(b)** infinitely many solutions, **(c)** a unique solution

$$\begin{aligned} 2x - y + 2az + t &= b \\ ay &= 1 - b \\ z + at &= -b \\ (a+1)t &= -2. \end{aligned}$$

3) Let $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$.

(a) Find A^{-1} by using elementary row operations.

(b) Find $\operatorname{adj}(A)$.

(c) Solve the system $Ax = b$ if $b = \begin{bmatrix} 6 \\ 7 \\ -2 \end{bmatrix}$.

4) Let $A = \begin{bmatrix} 1 & 2 & -1 & 2 \\ 2 & 5 & -1 & 3 \\ 3 & 6 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix}$. Compute the following determinants:

(a) $|A|$

(b) $|\frac{1}{2}A^{-1}|$

(c) $|\text{adj}(A)|$

(d) $|D|$ if $A \xrightarrow{R_2 \leftrightarrow R_3} B \xrightarrow{-50R_3+R_4 \to R_4} C \xrightarrow{-3R_1 \to R_1} D$

(e) $|C^3|$

(f) $|B^T D^{-1}|$

(g) $\left|\left(\frac{2}{3}D\right)^{-1}\right|$

5)

(a) Let $A = \begin{bmatrix} a & 0 & 0 \\ 2 & b & 0 \\ 3 & 1 & c \end{bmatrix} \times \begin{bmatrix} d & -1 & 3 \\ 0 & e & 1 \\ 0 & 0 & f \end{bmatrix}$

Suppose that $AX = 0$ has a nontrivial solution. Show that at least one of a, b, c, d, e, f must be zero.

(b) Show that if u_1 and u_2 are solutions of the linear system $Ax = b$, then $u_1 - u_2$ is a solution of the homogenous sytem $Ax = 0$.

(c) Given that $\begin{vmatrix} 1 & a & 2 \\ -1 & 1 & b \\ a & 2 & 3b \end{vmatrix} = 4$. Find $\begin{vmatrix} x & a+bx & 2 \\ -x & 1-bx & b \\ ax & 2+abx & 3b \end{vmatrix}$ in terms of x.